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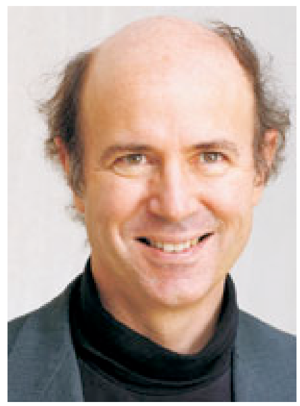
Whence the Force of *F* = *ma*? I: Culture Shock

Frank Wilczek



When I was a student, the subject that gave me the most trouble was classical mechanics. That always struck me as peculiar, because I had no trouble learning more advanced subjects, which were supposed to be harder. Now I think I've figured it out. It was a case of culture shock. Coming from mathematics, I was expecting an algorithm. Instead I encountered something quite different— a sort of culture, in fact. Let me explain.

Problems with *F* = *ma*

Newton's second law of motion, $F = ma$, is the soul of classical mechanics. Like other souls, it is insubstantial. The right–hand side is the product of two terms with profound meanings. Acceleration is a purely kinematical concept, defined in terms of space and time. Mass quite directly reflects basic measurable properties of bodies (weights, recoil velocities). The left–hand side, on the other hand, has no independent meaning. Yet clearly Newton's second law is full of meaning, by the highest standard: It proves itself useful in demanding situations. Splendid, unlikely looking bridges, like the Erasmus Bridge (known as the Swan of Rotterdam), do bear their loads; spacecraft do reach Saturn.

The paradox deepens when we consider force from the perspective of modern physics. In fact, the concept of force is conspicuously absent from our most advanced formulations of the basic laws. It doesn't appear in Schrödinger's equation, or in any reasonable formulation of quantum field theory, or in the foundations of general relativity. Astute observers commented on this trend to eliminate force even before the emergence of relativity and quantum mechanics.

In his 1895 *Dynamics*, the prominent physicist Peter G. Tait, who was a close friend and collaborator of Lord Kelvin and James Clerk Maxwell, wrote

"In all methods and systems which involve the idea of force there is a leaven of artificiality. . . . there is no necessity for the introduction of the word "force" nor of the sense–suggested ideas on which it was originally based."[1]

Particularly striking, since it is so characteristic and so over–the–top, is what Bertrand Russell had to say in his 1925 popularization of relativity for serious intellectuals, *The ABC of Relativity*:

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"If people were to learn to conceive the world in the new way, without the old notion of "force," it would alter not only their physical imagination, but probably also their morals and politics. . . . In the Newtonian theory of the solar system, the sun seems like a monarch whose behests the planets have to obey. In the Einsteinian world there is more individualism and less government than in the Newtonian."[2]

The 14th chapter of Russell's book is entitled "The Abolition of Force."

If $F = ma$ is formally empty, microscopically obscure, and maybe even morally suspect, what's the source of its undeniable power?

The culture of force

To track that source down, let's consider how the formula gets used.

A popular class of problems specifies a force and asks about the motion, or vice versa. These problems look like physics, but they are exercises in differential equations and geometry, thinly disguised. To make contact with physical reality, we have to make assertions about the forces that actually occur in the world. All kinds of assumptions get snuck in, often tacitly.

The zeroth law of motion, so basic to classical mechanics that Newton did not spell it out explicitly, is that mass is conserved. The mass of a body is supposed to be independent of its velocity and of any forces imposed on it; also total mass is neither created nor destroyed, but only redistributed, when bodies interact. Nowadays, of course, we know that none of that is quite true.

Newton's third law states that for every action there's an equal and opposite reaction. Also, we generally assume that forces do not depend on velocity. Neither of those assumptions is quite true either; for example, they fail for magnetic forces between charged particles.

When most textbooks come to discuss angular momentum, they introduce a fourth law, that forces between bodies are directed along the line that connects them. It is introduced in order to "prove" the conservation of angular momentum. But this fourth law isn't true at all for molecular forces.

Other assumptions get introduced when we bring in forces of constraint, and friction.

I won't belabor the point further. To anyone who reflects on it, it soon becomes clear that $F = ma$ by itself does not provide an algorithm for constructing the mechanics of the world. The equation is more like a common language, in which different useful insights about the mechanics of the world can be expressed. To put it another way, there is a whole culture involved in the interpretation of the symbols. When we learn mechanics, we have to see lots of worked examples to grasp properly what force really means. It is not just a matter of building up skill by practice; rather, we are imbibing a tacit culture of working assumptions. Failure to appreciate this is what got me in trouble.

The historical development of mechanics reflected a similar learning process. Isaac Newton scored his greatest and most complete success in planetary astronomy, when he discovered that a single force of quite a simple form dominates the story. His attempts to describe the mechanics of extended bodies and fluids in the second book of *The Principia* were path breaking but not definitive, and he hardly touched the more practical side of mechanics. Later physicists and mathematicians including notably Jean d'Alembert (constraint and contact forces), Charles Coulomb (friction), and Leonhard Euler (rigid, elastic, and fluid bodies) made fundamental contributions to what we now comprehend in the culture of force.

Physical, psychological origins

Many of the insights embedded in the culture of force, as we've seen, aren't completely correct. Moreover, what we now think are more correct versions of the laws of physics won't fit into its language easily, if at all. The situation begs for two probing questions: How can this culture continue to flourish? Why did it emerge in the first place?

For the behavior of matter, we now have extremely complete and accurate laws that in principle cover the range of phenomena addressed in classical mechanics and, of course, much more. Quantum electrodynamics (QED) and quantum chromodynamics (QCD) provide the basic laws for building up material bodies and the nongravitational forces between them, and general relativity gives us a magnificent account of gravity. Looking down from this exalted vantage point, we can get a clear perspective on the territory and boundaries of the culture of force.

Compared to earlier ideas, the modern theory of matter, which really only emerged during the 20th century, is much more specific and prescriptive. To put it plainly, you have much less freedom in interpreting the symbols. The equations of QED and QCD form a closed logical system: They inform you what bodies can be produced at the same time as they prescribe their behavior; they govern your measuring devices — and you, too!— thereby defining what questions are well posed physically; and they provide answers to such questions — or at least algorithms to arrive at the answers. (I'm well aware that QED + QCD is not a complete theory of nature, and that, in practice, we can't solve the equations very well.) Paradoxically, there is much less interpretation, less culture involved in the foundations of modern physics than in earlier, less complete syntheses. The equations really do speak for themselves: They are algorithmic.

By comparison to modern foundational physics, the culture of force is vaguely defined, limited in scope, and approximate. Nevertheless it survives the competition, and continues to flourish, for one overwhelmingly good reason: It is much easier to work with. We really do not want to be picking our way through a vast Hilbert space, regularizing and renormalizing ultraviolet divergences as we go, then analytically continuing Euclidean Green's functions defined by a limiting procedure, . . . working to discover nuclei that clothe themselves with electrons to make atoms that bind together to make solids, . . . all to describe the collision of two billiard balls. That would be lunacy similar in spirit to, but worse than, trying to do computer graphics from scratch, in machine code, without the benefit of an operating system. The analogy seems apt: Force is a flexible construct in a high–level language, which, by shielding us from irrelevant details, allows us to do elaborate applications relatively painlessly.

Why is it possible to encapsulate the complicated deep structure of matter? The answer is that matter ordinarily relaxes to a stable internal state, with high energetic or entropic barriers to excitation of all but a few degrees of freedom. We can focus our attention on those few effective degrees of freedom; the rest just supply the stage for the actors.

While force itself does not appear in the foundational equations of modern physics, energy and momentum certainly do, and force is very closely related to them: Roughly speaking, it's the space derivative of the former and the time derivative of the latter (and $F = ma$ just states the consistency of those definitions!). So the concept of force is not quite so far removed from modern foundations as Tait and Russell insinuate: It may be gratuitous, but it is not bizarre. Without changing the content of classical mechanics, we can cast it in Lagrangian terms, wherein force no longer appears as a primary concept. But that's really a technicality; the deeper questions remains: What aspects of fundamentals does the *culture* of force reflect? What approximations lead to it?

Some kind of approximate, truncated description of the dynamics of matter is both desirable and feasible because it is easier to use and focuses on the relevant. To explain the rough validity and origin of specific concepts and idealizations that constitute the culture of force, however, we must consider their detailed content. A proper answer, like the culture of force itself, must be both complicated and open–ended. The molecular explanation of friction is still very much a research topic, for example. I'll discuss some of the simpler aspects, addressing the issues raised above, in my next column, before drawing some larger conclusions.

Here I conclude with some remarks on the psychological question, why force was— and usually still is— introduced in the foundations of mechanics, when from a logical point of view energy would serve at least equally well, and arguably better. The fact that changes in momentum— which correspond, by definition, to forces— are visible, whereas changes in energy often are not, is certainly a major factor. Another is that, as active participants in statics— for example, when we hold up a weight— we definitely feel we are doing something, even though no mechanical work is performed. Force is an abstraction of this sensory experience of exertion. D'Alembert's substitute, the virtual work done in response to small displacements, is harder to relate to. (Though ironically it is a sort of virtual work, continually made real, that explains our exertions. When we hold a weight steady, individual muscle fibers contract in response to feedback signals they get from spindles; the spindles sense small displacements, which must get compensated before they grow.[4]) Similar reasons may explain why Newton used force. A big part of the explanation for its continued use is no doubt (intellectual) inertia.

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