REFERENCE FRAME

Mass without Mass I: Most of Matter

Frank Wilczek

With his unique talent for the paradoxical profundity, John Wheeler coined the phrase "mass without mass" to advertise the goal of removing any mention of mass from the basic equations of physics.[1] Can we really hope to do this? How far have we come? Why should we try? In this piece, I answer the first question and part of the second; in my next column, I'll round out the story and look ahead.

As commonly used, the words "massive" and "weighty" connote things that are too obvious and significant to ignore, as in a massive fraud or a weighty opinion. Thus our very language conditions us to think of the mass of a physical object as one of its primary characteristics. So does our everyday experience, and even our early education in physics. Indeed, the concept of mass lies at the heart of Newtonian physics. It appears explicitly both in the foundational equation $F = ma$ and in the law of universal gravitation $F = GMm/r^2$.

Later developments in physics made the concept of mass seem less irreducible, and less basic. This undermining process started in earnest with the theories of relativity. The famous equation $E = mc^2$ of special relativity theory, written that way, betrays the prejudice that we should express energy in terms of mass. But it doesn't take an Einstein to derive from that equation $m = E/c^2$, which suggests the possibility of explaining mass in terms of energy. And the conceptual hub of the general theory of relativity, the equivalence principle, is the observation that the response of a body to gravitation is independent of its mass. Consistent with this observation, Newton's two laws can be combined into $a = GM/r^2$, wherein m does not appear. The central equation of general relativity theory,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

(in appropriate units), equates the curvature of spacetime to the energy–momentum of matter. Einstein referred to the left-hand side as a palace of gold, and to the right-hand side as a hovel of wood, thus expressing his ambition to make improvements on the right-hand side, to root it in concepts of depth and beauty comparable to Riemannian geometry. Of course, it is only on the right-hand, wooden side that masses of particles occur, raw and unadorned. Can we replace them with finer material?

Quantum field theory greatly simplifies our task by vastly reducing the inventory of different parts we need to replace. In quantum field theory, the primary elements of reality are not individual particles, but underlying fields. Thus, for example, all electrons are but excitations of an underlying field, naturally called the electron field, which fills all space and time. This formulation explains why all electrons everywhere and for all time have exactly the same properties, including, of course, the same mass. If one constructs all matter from excitations of a few fields, as we do in the modern Standard Model, the challenge of mass takes a new and profoundly simpler form. At worst, we will have to specify a few numerical parameters—one for each fundamental field—to account for mass in general.

In practice, we do much better. The bulk of the mass of ordinary matter (better than 99%) comes from the masses of protons and neutrons. In quantum chromodynamics (QCD), the protons and neutrons appear as secondary, composite structures built up from quarks and gluons. We can maintain an excellent approximation to reality while working with a truncated version of QCD, which contains only the color gluons plus up and down quark fields. The heavier quarks play an extremely minor role in the structure of the proton and neutron.

Our theory of the color gluons is derived from a powerful symmetry principle—non-Abelian, or Yang–Mills, gauge symmetry—similar in many respects to the general covariance of general relativity. Gauge symmetry forbids mass terms for the gluon fields. Thus color gluons, like gravitons and photons, and for similar reasons, have no mass. Furthermore, there is much phenomenological evidence that the mass terms associated with up and down quarks are quite small. Let us set them to zero. Now our resulting truncated, approximate version of QCD contains no mass terms at all. (In a genuine sense, it has no free parameters whatsoever.[2]) Yet, if we use it to calculate the mass of protons and neutrons—the mass of ordinary matter—we find it is accurate[3] to within 10%!

How is it possible that massive protons and neutrons can be built up out of strictly massless quarks and gluons? The key is $m = E/c^2$. There is energy stored in the motion of the quarks, and energy in the color gluon fields that connect them. This bundling of energy makes the proton's mass.

The emergent picture of the proton mass realizes, in a different context, a modified form of the dream of Hendrik A. Lorentz[4] (pursued by many others including Henri Poincaré, P. A. M. Dirac, Wheeler, and Richard Feynman) to account for the electron's mass entirely in terms of its electromagnetic fields. A classical point electron is surrounded by an electron field varying as $1/r^2$. The energy in this field is infinite, due to a divergent contribution around $r \to 0$. Lorentz hoped that in a correct model of electrons, they would emerge as extended objects, and that the energy in the Coulomb field would come out finite and in fact account for all or most of the inertia of electrons.

Later progress in the quantum

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theory of electrons rendered this program moot, by showing that the charge of an electron—and therefore, of course, the singularity of its associated electric field—is intrinsically smeared out by quantum fluctuations in the electron's position. As a result, the electric field energy of an electron makes only a small correction to its total mass. Thus Lorentz's dream, in its original form, is not realized. But beautiful ideas are rarely entirely wrong, and something close to Lorentz's idea is embodied in modern QCD. Quarks carry color charge, and generate color electric fields analogous to the ordinary electric fields around electrons. The potentially diverging energy of color electric fields close to the quark is removed by quantum mechanics, just as for ordinary electric fields around electrons. But unlike ordinary electric fields, color electric fields do not automatically fall off rapidly far from their source. Indeed, the color electric field energy generated by an isolated quark is calculated to be truly infinite, due to the energy it creates in distant fields. This property explains why quarks are never observed in isolation.

Triples of quarks, however, can cunningly contrive to generate fields that cancel at very large distances. To build protons and neutrons, they must do this. Even so, at finite distances the fields do not cancel exactly, and so a finite field energy remains. According to QCD, it is precisely this color field energy that mostly makes us weigh. It thus provides, quite literally, "mass without mass."

Thus QCD takes us a long stride toward the Einstein–Wheeler ideal of "mass without mass." For ordinary matter, *quantitatively*, it brings us amazingly close. If your friend puts on a few pounds yet complains, "But I never eat anything heavy," modern physics sanctions you to give her (or him) the benefit of the doubt.

References

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4. H. A. Lorentz, Proc. Acad. Sci., Amsterdam **6** (1904), reprinted in A. Einstein *et al.*, *The Principle of Relativity*, Dover, New York (1952), p. 24. See also, especially, R. P. Feynman, *Lectures in Physics*, vol. 2, Addison-Wesley, Reading, Mass. (1964), chap. 28. ■